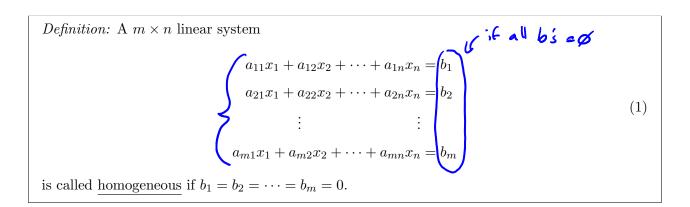
Homogeneous Linear Systems



Concept: All homogeneous linear systems have the trivial solution

$$x_1 = 0, \qquad x_2 = 0, \qquad \cdots, \qquad x_n = 0$$

and are therefore consistent.

Example 8: Show that the underdetermined homogeneous linear system with the given augmented matrix has infinitely many solutions.

$$\begin{bmatrix} 1 & 3 & 1 & 0 & 0 \\ 3 & -1 & -3 & -4 & 0 \\ 2 & 1 & -3 & 0 & 0 \end{bmatrix}$$
(2)

$$R_{3} := R_{3} + (0R_{2} \begin{bmatrix} 1 & 3 & 1 & 0 & 0 \\ 3 & -1 & -3 & -4 & 0 \\ 2 & 1 & -3 & 0 & 0 \end{bmatrix} \begin{pmatrix} R_{3} := R_{3} - 2R_{1} \\ R_{2} := R_{3} - 2R_{1} \\ R_{3} := R_{3} - 2R_{1} \\ R$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \times_{y=t} - \text{free var}$$

$$\chi_{3} - t = 0 \Rightarrow \chi_{3} = t$$

$$\chi_{2} + t = 0 \Rightarrow \chi_{2} = -t$$

$$\chi_{1} - 2t = 0 \Rightarrow \chi_{1} = 2t$$

$$\chi_{1} = 2t$$

$$\chi_{1} = t$$

$$\chi_{2} = t$$

$$\chi_{2} = t$$

$$\chi_{3} = t$$

$$\chi_{4} = t$$

Theorem 3 (Poole 2.3): Any underdetermined $m \times n$ homogeneous linear system (n > m) has infinitely many solutions.